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SOME ASPECTS OF THE PROBLEM OF DETERMINATION OF THE TRUE RADIO BRIGHTNESS DISTRIBUTION OVER THE SKY

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SUMMARY

It is shown that the problem of searching for the true distribution of radio brightness over the sky amounts to the solution of the integral equation (1) below only for a specific operational regime of the radio telescope. Estimates are given of the averaging time of radio-telescope's output signal assuring the detection of radio sources of various magnitudes. The possibility of solving the problem of reduction by an analogous method is discussed. It allows to reduce the processing of radio brightness, taken down by a special method, to the passing of the signal, proportional to this distribution function, through a linear filter.

* * *

1. The determination of the true distribution of radio emission over the sky constitutes one of the important problems of radio astronomy. The basic idea behind all the methods of finding the true distribution of radio brightness considered in literature is that of reversibility of the transformation, materialized by the antenna, which is expressed in the unidimensional case by the equation

$$f(x_0)=\int_{-\infty}^{\infty} g(x-x_0)\,\varphi(x)\,dx. \qquad (1)$$

Here f (x) is the measurable distribution of the intensity of radio emission by the angular coordinate x, g (x) is the antenna's radiation pattern by power, φ (x) is the true distribution of intensity. It is

* NEOTORYYE ASPEKTY ZADACHI OPREDELENIYA ISTINNOGO RASPREDELENIYA YARKOSTI PO NEBU

assumed that the antenna is linear and that the signals are incoherent.

The entire variety of methods of finding $\varphi(x)$ is linked with different methods of resolving the equation (1) and various means for their realization. The main attention is usually given to the precision of calculations. At the same time it is assumed that the measured distribution of the power of $f(x)$ is obtained with a sufficient precision and that the connection between $f(x)$ and $\varphi(x)$ is expressed by the equation (1).

Let us now discuss the conditions that have to be fulfilled in order to satisfy these assumptions.

In the experiment the distribution of power by the angle is obtained by way of averaging the square of antenna's output signal. This procedure is obviously expressed by the correlation

$$f(x_0, t_0) = \int_{-\infty}^{t_0} B(t - t_0) \int_{-\pi}^{\pi} g[x - x_0(t)] s^2(x, t)\, dx\, dt, \qquad (2)$$

where $B(\tau)$ is the pulse response of the neutralizing system, $s^2(x, t)$ is the square of signal realization, proceeding from the source with an angular coordinate x, $x_0(t)$ is a function, describing the displacement of the axis of antenna pattern with time, t_0 is the moment of time at which the function $\underline{f}$ is measured, x_0 is the angular coordinate, to which the measured value of the function $\underline{f}$ is ascribed.

It is easy to see that (2) passes to (1) only at fulfillment of certain, quite specific conditions. First of all this is an obvious condition for a "sufficient narrowness" of the pattern and the absence of sources near the boundaries of the sector considered; it allows the passing to infinite limits in the integral over $\underline{x}$. Secondly, the antenna pattern must be fixed relative to emission sources during the entire averaging time, that is, $x_0(t)$ must be a constant; only then can (2) be rewritten in a form close to (1):

$$f(x_0, t_0) = \int_{-\infty}^{\infty} g(x - x_0) \left[\int_{-\infty}^{t_0} B(t_0 - t) s^2(x, t)\, dt \right] dx. \qquad (3)$$

Finally, in order that the expression in brackets can be substituted by the true distribution of $\varphi(x)$, the function $B(\tau)$ must also satisfy specific conditions (see part 2).

Therefore, contrary to the established opinion, the bringing up of the reduction problem to the solution of the equation (1) is, strictly speaking, possible only for a specific operating regime of the entire system, when

1) the scanning is discrete; during the necessary time interval of T averaging, at the end of which the reading of the measured function f (x) is taken down, the antenna remains fixed relative to the source (the necessary value of T is estimated in part 2);

2) at the end of the interval of T the antenna axis is displaced by a certain angle Ω and stops again for T seconds in order to take down the following reading; then the procedure is repeated, so long as the given sector is not covered (the value of Ω is linked with the system's resolution; the parameters of the system, affecting Ω, are discussed in the part 4).

The question as to what the reduction precision is when obtaining the function f (x) by way of continuous scanning and the subsequent formal substitution into the equation (1), remains open. However, certain researchers use precisely this procedure (see references at the end of the paper). That is why the indicated question requires a special consideration, which is beyond the object of the present paper. We may only state, that in this case the equation (2) and not (1) must be the initial one for the analysis.

2. - Assume that the conditions, for which the experiment is described by the equation (3), are fulfilled. It is necessary to estimate quantitatively the conditions, for which the equation (3) is reduced to (1). As a matter of fact the readings of the device, measuring the power of radio emission, is a random quantity, which approaches the true value searched for only at limit (at averaging time increase). In order to assure the measurement of power with a guaranteed precision, one may estimate the averaging time necessary to that effect. It is important to note that even in the absence of noises this time depends on the range of values of the power of sources.

Assume that the emission of radio sources is subject to standard distribution. Assume further that the power P of the emission is estimated by way of averaging N readings of squares of the received signal's amplitude. This means, that the weight function B (τ) constitutes a rectangular pulse of duration T, to which correspond N readings:

$$\int_{-\infty}^{t_0} B(t_0-t)s^2(x, t)\,dt=\frac{1}{T}\int_{t_0-T}^{t_0} s^2(x, t)\,dt=\frac{1}{N}\sum_{t_0-N}^{t_0} s^2(x, t_k). \qquad (4)$$

It is well known [1] that the standard deflection of the measured power then is

$$\sigma_p=\sqrt{2/N}\,P. \qquad (5)$$

In order to reliably reveal a weak source against the background of strong source's intensity fluctuations, and also with the view of avoiding that the fluctuations of a strong source's power subsequently provide fictitious "weak sources", it is required that $\sigma_{P_{max}}$ be less than P_{min} by a given number n of times, where P_{min} and P_{max} are the anticipated powers of, respectively the weakest and the strongest sources of radio emission:

$$P_{min}/\sigma_{P_{max}}=n. \qquad (6)$$

Assume further as given the dynamic range of radiotelescope's receiver, that is, the ratio between the minimum and maximum possible signals is known :

$$P_{max}/P_{min}=m. \qquad (7)$$

From formulas (5) — (7) it follows that the number of readings required for the averaging is :

$$N=2(mn)^2. \qquad (8)$$

On the other hand, N is related to receiver pass band and averaging time. Indeed, in the case when the readings are uncorrelated, we have for the uniform radio emission spectrum in the receiver's band

$$N=2FT, \qquad (9)$$

where F is the receiver's band and T the averaging time. Hence it follows that the required averaging time is

$$T=\frac{1}{F}(mn)^2. \qquad (10)$$

According to (10), at band expansion we are either compelled to increase the receiver's band or the averaging time.

If we take requirements, that are not too much overrated, and for which $n = 10 \div 100$, $m = 10 \div 1000$, we have

$$T = \frac{1}{F} (10^4 \div 10^{10}) \text{ sec} \qquad (11)$$

For example, depending upon m and n, the quantity T may vary from 10^{-3} to 10^3 sec for a band $F = 10^7$ cps. This example may serve as a basis for an approximate conclusion of the fact, that inasmuch as $T \sim 10^{-3}$ is a sufficiently small time interval, the use of the equation (1) at substitution in it of the function $f(x)$, obtained at continuous scanning, can provide a sufficiently accurate expression for $\varphi(x)$ only at fairly low requirements for m and n. It is also necessary to realize that even the "slowness" of scanning at uniform distribution of power by the angle and at great m and n does not save the situation. In order to ascertain the above it is sufficient to conduct the computations of the unsteadiness of the sector antenna's output signal.

3.- To conduct calculations within an analogous computation mode certain physical phenomena, described by the corresponding mathematical operation, are fairly often applied. Such an approach can be also attempted in the case under consideration. If we assume that the variable x in the equation (1) characterizes the time, this equation will describe the passage of the signal $\varphi(x)$ through the linear filter with response of $g(x)$, as a result of which a signal $f(x)$ will be obtained at filter output. It is thus natural to let pass a signal proportional to a well known measured function $f(x)$ through the filter materializing the inverse transformation if we desire to obtain the function $\varphi(x)$ searched for. Therefore, instead of coupling the radio telescope with a computer, effecting cumbersome calculations, it is sufficient to combine the receiver with a sufficiently simple linear filter. If the characteristic of the direct filter is $G(i\omega)$, that of the inverse filter must be expressed by the function $G^{-1}(i\omega)$. The Peytli-Viner* criterion of [8] asserts that the characteristic of a physically realizable filter must satisfy the condition

$$\int_{-\infty}^{\infty} \frac{|\log G(i\omega)|}{1+\omega^2}\, d\omega < \infty. \qquad (12)$$

* in transliteration

Hence it follows immediately that there exists for every physically realizable filter, an inverse filter to it, just as equally realizable. Indeed, $|\lg G^{-1}(i\omega)| = |\lg G(i\omega)|$.

Subsequently, the problem is formulated as follows : are there such antenna radiation patterns that would correspond to physically realizable filters ? In order to show the existence of patterns not satisfying this requirement, it is sufficient to take a circular, nondirectional pattern. It follows from general considerations that the pattern, required by us, must be nonzero in a limited interval of angles, say less than 2π, and, to the extent possible, substantially lesser intervals. A simple example of such a pattern is provided by the sectorial pattern. Therefore, the question is in fact reduced to the problem of the possibility of creating a linear antenna, with a pattern of the given class of functions.

Let us point to still another possibility of practical realization of the inverse filter (besides its synthesis by direct methods). This possibility is based upon a well known method in analog computing techniques denoted as the method of the inverse function. At times it is much simpler to synthetize the direct filter. Then the inverse filter may be obtained by switching the direct filter onto the amplifier feedback with a high amplification factor. Thus, the switching in of the direct filter, whose response reflects the antenna pattern, onto the amplifier feedback, will provide the corresponding inverse filter.

4.- When taking into account the influence of external and instrumental noises on the accuracy of the solution of the problem under consideration, two independent problems arise. The first consists in the accounting of background, antenna, receiving apparatus and of the power-measuring circuit noises. These noises will lead to the appearance of a certain additional adjunct constant (equal to the power of these noises), which may be easily eliminated ahead of the inverse filter input. In reality, when accounting of these noises, we have instead of (3)

$$f_1(x_0, t_0) = f(x_0, t_0) + \int_{-\infty}^{t_0} B(t_0 - t) n^2(t)\, dt = f(x_0, t_0) + c\,\sigma_{ш}^2. \qquad (13)$$

Here n (t) is the realization of the aggregate noise to the power measurement circuit. However, after deduction of the indicated constant component the fluctuations will remain owing to finiteness of averaging time. Let us estimate the magnitude of these fluctuations. According to (5), the standard deflection of the fluctuations is :

$$\beta_ш = \sqrt{2/N},\ \sigma^2_ш = \sigma^2_ш/mn,$$

where N defines the averaging time.

At passing to the last equality formula (8) was taken into account. If the aggregate power of noises is equal to the minimum power of the useful source ($\sigma^2_ш = P_{\min}$), $\beta_ш$ is by <u>mn</u> times smaller than the minimum rejection of the measured function (that is, by several orders in practice).*

Thus, the accounting of noises superimposed on the useful signal to the power measurement circuit, does not lead to serious complications.

The other, quite essential problem is the accounting of apparatus' noises between the power measurement circuit and the inverse filter (these are noises from linear amplifiers, which may stand in front of the inverse filter, and the noises from components of the inverse filter itself). Inasmuch as the characteristic of the inverse filter rises infinitely as the characteristic of the direct filter decreases, a natural doubt arises, as to whether or not these noises may bring to zero the entire effect of the inverse filtration. The narrowing of the inverse filter's band, while limiting the power of the output noise, limits simultaneously also the resolution of the sources when the angular distances between them are sufficiently small. Thus the consideration of this question from the quantitative viewpoint makes sense.

Since the reversal circuit is linear, we may consider the action upon by the useful and noise components separately. Assume, for instance, that the filter response, equivalent to the antenna, is characterized by the Gauss curve. Then the measured distribution of intensity, corresponding to the point source, will be determined by the function

$$f(t) = KPe^{-\beta^2 t^2}. \qquad (14)$$

Here K is the signal amplification factor, when produced by the power measurement circuit, P is the source's intensity, multiplied by the maximum value of the antenna's radiation pattern. β is a factor, determined

* Incidentally, at weak requirements for m and n (and this allows precisely

../.. [next page]

by the width of the pattern and the rate of function's f (t) readout; $\beta = 0{,}8\,t_0^{-1}$, where t_0 is the pulse width of f (t), measured by the half-level. The spectrum of the function (14) is determined by the correlation

$$F_f(\omega) = P\,\frac{\sqrt{\pi}}{\beta}\,e^{-\omega^2/4\beta^2} \qquad (15)$$

and, consequently, the charactristic of the inverse filter is described by the function

$$G(\omega) = C e^{\omega^2/4\beta^2} \qquad (16)$$

where $C > 0$ is the filter's transmission factor at zero frequency.

Desirous to limit the noise power at reversal circuit output, we limit the frequency band $[0, \Omega]$, in which the characteristic of the real filter coincides with (16) (outside this interval it is zero). Then, the power of the useful component at inverse filter output is

$$S = \int_0^{\Omega} F_f^2(\omega)\,G^2(\omega)\,d\omega = \pi\left(\frac{KCP}{\beta}\right)^2 \Omega. \qquad (17)$$

At the same time to the δ – shaped real distribution function of power corresponds a function of the type sin x/x, the width of which is determined by the quantity π/Ω (along the intersection with the axis $\varphi = 0$). Therefore, the quantity Ω is linked with the resolving power of the entire system.

Let us now estimate the noise dispersion at output of the approximate inverse filter with a limited band. Assume, for instance, that the noises of the amplifier N_1 and of inverse filter's component N_2 have uniform spectra, so that the aggregate spectrum of noise power is expressed by the function

$$N(\omega) = K^2 N_1 + N_2 = N_0. \qquad (18)$$

Then the power of the noise component at inverse filter output is

$$N = N_0 \int_0^{\Omega} G^2(\omega)\,d\omega = \sqrt{2}\,N_0 C^2 \beta M\left(\frac{\Omega}{\sqrt{2}\,\beta}\right), \qquad (19)$$

where $M(x) = \int_0^x e^{t^2}\,dt$

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* [continued from page 7].. to use equation (1) in case of continuous scanning), the effect of noises will become substantial.

The signal-to-noise ratio at inverse filter output is

$$\frac{S}{N} = \pi \frac{(KP/\beta)^2}{K^2N_1+N_2} D_1\left(\frac{\Omega}{\sqrt{2}\beta}\right), \tag{20}$$

where $D_1(x) = x/M(x)$.

Cases of nonuniform noise spectrum are encountered in practice. If, for example, the noise spectrum has the form $N(\omega) = N_0\omega^{-\alpha}$, the integral (19) is reduced to the expression

$$N = C^2N_0\int_0^{\Omega} e^{\omega^2/4\beta^2}\omega^{-\alpha}\,d\omega =$$

$$= \frac{C^2N_0}{2} B\left(1, 1-\frac{\alpha+1}{2}\right) {}_1F_1\left(1-\frac{\alpha+1}{2};\ 2-\frac{\alpha+1}{2};\ \left(\frac{\Omega}{2\beta}\right)^2\right). \tag{21}$$

Hence we obtain that the signal-to-noise ratio is

$$\frac{S}{N} = \pi \frac{(KP)^2}{N_0\beta} D_2\left(\frac{\Omega}{2\beta}\right), \tag{22}$$

where $D_2 = (\Omega/2\beta)B^{-1}{}_1F_1^{-1}$ and the arguments at functions B and ${}_1F_1$ are the same as in (21).

The most characteristic (and the most promising) is the circumstance that (20) and (22) consist of two comultipliers, which in the experiment may be preassigned independently. The functions D (x) are only determined by how much we wish to increase the resolution of two close sources. The functions D (x) decrease rapidly, so that at increase of requirements for the resolution, other conditions being equal, the ratio S / N drops. However, at constant degree of resolving power capability (that is, at constant ratio Ω/β we have the possibility of increasing S / N independently at the expense of the first comultiplier. Quite promising appears to be the decrease of the factor β, which, from the physical standpoint, corresponds to a slowed-down reproduction of the function f (x), fed at input of the inverse filter; at the same time, from the theoretical viewpoint, no limitations of any kind arise here.

In spite of the fact that we discussed here only the concrete form of the function (14), it is physically clear that the conclusions will remain valid for any form of that function (provided it corresponds to the realized filter only), for the effect of S / N increase at the expense of the first comultiplier is based upon an equivalent narrowing of the frequency band. The latter is possible at a constant factor of resolving power capability increase of the system at the expense of the slowed-down reproduction of the function $f(x)$.

Therefore, it is possible to obtain the required resolution of close sources for any given signal-to-noise ratio at inverse filter output. This constitutes a strong argument in favor of the indicated method for resolving the reduction problem. The only complication here is the requirement of preliminary writing of the function $f(x)$, and then of its reproduction at the necessary tempo prior to feeding the inverse filter.

*** THE END ***

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